Physical Foundations for the Development of Control Systems for Avoidance of undesired Casting of Shadows from Wind Energy Converters on appointed Objects

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Abstract:

In some cases the casting of shadows from the rotating blades of a Wind-Energy-Converter can cause problems for someone's living or working conditions because of the flickering daylight. Aim of this paper is to work out the theoretical physical foundations which enables us to develop a control system, which avoids an overlap of the rotating shadow and a restricted area. It is possible to calculate the time dependent shadow curvature in advance.

Introduction:

To reach this aim, the mathematical formulas describing the position of the sun relative to a Wind-Energy-Converter were combined with the projective mappings of the pole and rotator circle. With the help of the parameters like geographic position, month, day, time and the orientation of the blades the worked out formulas enable us to calculate the shadow region for every moment. This information could be used by a control system to protect an appointed area from flickering daylight.

Physical Foundations:

Imagine a Wind-Energy-Converter with the pole height H relative to the appointed object which has to be protected from shadowing as demonstrated in Figure 5. Set this into a coordinate system as shown in Figure 2. For the following considerations it is sufficient to restrict our derivations to a horizontal plane, with respect to topographic maps. The sun with an altitude h (→ Figure 1) produces a shadow of the pole as well as of the rotor blades. The shadow of the rotor hub has the coordinates

$$\left(S_x, S_y\right) = S\left(\sin a, \cos a\right) \quad \text{with} \quad S = \frac{H}{\tan(h)} \quad . \tag{1}$$

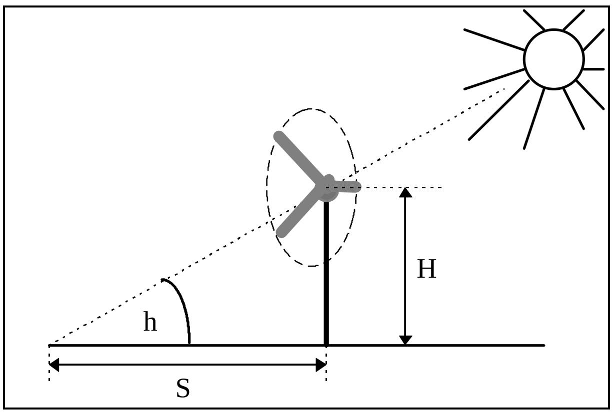

Figure 1: Sun's altitude and shadow

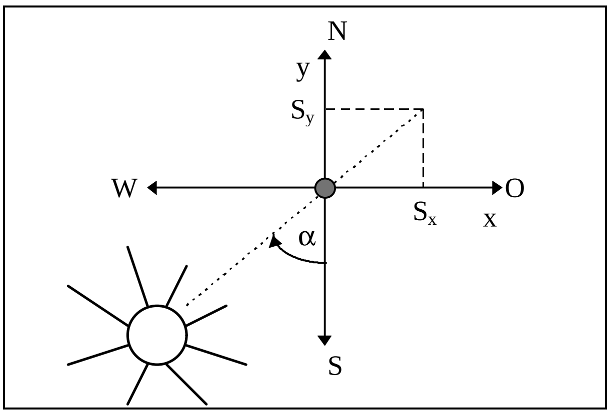

Figure 2: Sun's azimuth and coordinate system

Similar to calculations of the solar insolation for Photovoltaic Systems /1/ we use the following interrelations /2/, /3/:

Sun's altitude:

$$h = \arcsin\left(-\cos j \cdot \cos x \cdot \cos d + \sin d \cdot \sin j\right) \;, \qquad 0° \le h \le 90° \qquad (2)$$

$j \equiv$ latitude , $x \equiv$ hour angle , $d \equiv$ declination of the sun

and

$$h_{korr} = h + \frac{K_1}{K_2 + h} - K_3 \;, \qquad -0{,}5817° \le h \le 90° \qquad (3)$$

with respect to the atmospheric diffraction

$K_1 = 1{,}4705°$, $K_2 = 3{,}0427°$, $K_3 = 0{,}0158°$

Sun's azimuth:

$$a = \arctan\frac{e_y}{e_x} \;, \; -180° \le a \le 180° \qquad (4)$$

with

$$e_x = -\cos d \cdot \sin j \cdot \cos x - \sin d \cdot \cos j \;, \qquad e_y = -\cos d \cdot \sin x \qquad (5)$$

and

quadrant	I	II	III	IV
α	α	π + α	α - π	α

(with respect to the signs of e_x and e_y)

Sun's declination:

$$d = \frac{a_0}{2} + \sum_{i=1}^{3} a_i \cos\left(iN\frac{2p}{365} + b_i\right) \qquad (6)$$

with

$a_0 = 0{,}7896°$, $a_1 = -23{,}2559°$, $a_2 = -0{,}3915°$, $a_3 = -0{,}1764°$,

$b_1 = 0{,}1582$, $b_2 = 0{,}0934$, $b_3 = 0{,}4539$

$N = INT\left(30M + 0{,}6 \cdot |M - 3| - 30{,}5\right) + D$ for $M \ne 2$,

$N = 31 + D$ for $M = 2$

$M \equiv$ Month , $D \equiv$ Day

Hour angle:

$$x = LT \cdot w \;, \qquad LT = CET + TEQ - \frac{1}{w}\left(l_0 - l\right) \text{ with } w = \frac{360°}{24h} \;, \qquad (7)$$

$LT \equiv$ Local Time, $CET \equiv$ Central European Time,

$l \equiv$ Longitude, $l_0 \equiv 15°$ for CET ,

$TEQ \equiv$ Time Equation

Time Equation:

$$TEQ = \frac{a_0}{2} + \sum_{i=1}^{3} a_i \cos\left(iN\Omega + b_i\right) \;, \; \Omega = \frac{2p}{365} \text{ resp. } \Omega = \frac{2p}{366} \qquad (8)$$

with

$a_0 = 0{,}0132$ min , $a_1 = 7{,}3525$ min , $a_2 = 9{,}9359$ min , $a_3 = 0{,}3387$ min ,

$b_1 = 1{,}4989$, $b_2 = 1{,}9006$, $b_3 = 1{,}8360$

The Local Time for sunrise and sunset is given by the formula

$$t = \frac{1}{w} \arccos \frac{\sin d \cdot \sin j + 0{,}010153}{\cos d \cdot \cos j} . \qquad (9)$$

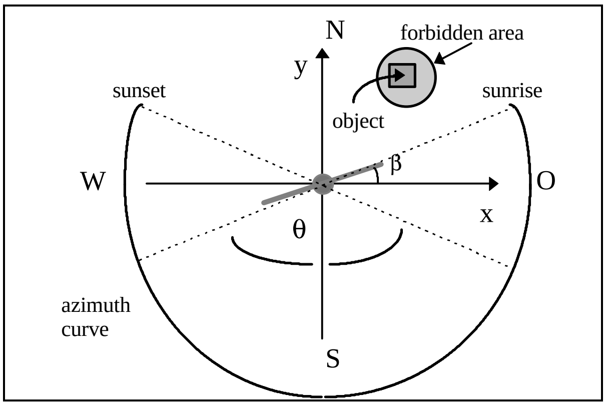

Figure 3: Appointed Object with 'forbidden' circle

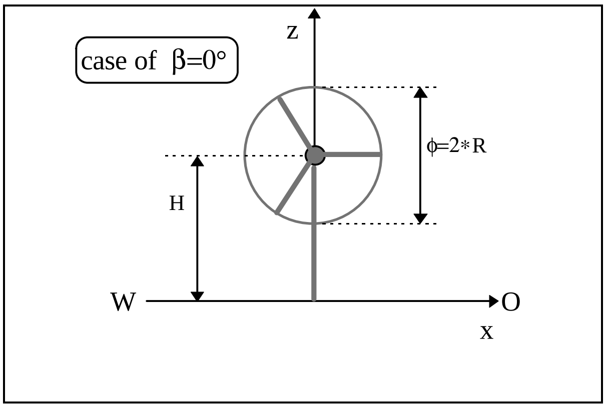

Figure 4: Rotator Circle

Let β be the angle between the rotor circle and the x-axis, which is equivalent to the deviation of its surface normal to the north. The coordinates of the circle curvature are

$$(x_R, y_R, z_R) = (0,0,H) + R \cdot \underline{\underline{D}}(b)(\cos t_R, 0, \sin t_R)^T \qquad (10)$$

with

$t_R \in [0,2p]$ and $\underline{\underline{D}}(b)$ the matrix of rotation relative to the z-axis :

$$\underline{\underline{D}}(b) = \begin{pmatrix} \cos b & -\sin b & 0 \\ \sin b & \cos b & 0 \\ 0 & 0 & 1 \end{pmatrix} .$$

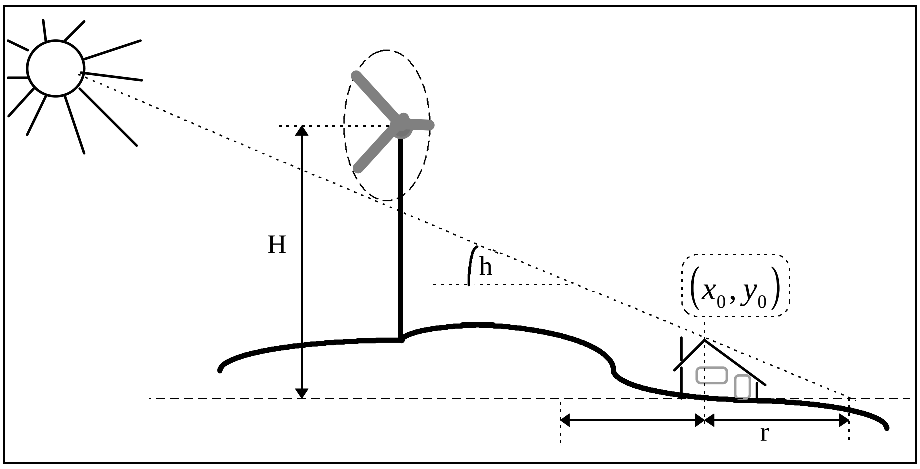

Figure 5: Geometric correlation

The coordinates of the 'forbidden' circle are $(X,Y,0) \rightarrow (X,Y)$ with

$$(X,Y) = (x_0, y_0) + r \cdot (\cos t, \sin t),\ t \in [0,2p], \qquad (11)$$

centre (x,y) and radius r.

To find a criterion to determine the minimum radius $r = r_{\min}$ which is necessary to be sure that the shadows of the rotating blades will not cover the object, we assume, that the sun, the rotor hub and the top of the object are all in one plane as shown in figure 5. Then there exists a minimum sun altitude $h = h_r$, so that for angles $h > h_r$ the shadow of the rotator would possibly reach the object. If the altitude is deeper than h_r the shadow would reach a region behind the forbidden area, in this case it is therefore impossible for the wind-energy-converter to cast a shadow on the object.

The altitude h_r is given by

$$\tan(h_r) = \frac{H - R - H_{object}}{\sqrt{x_0^2 + y_0^2}} = \frac{H_{object}}{r_{\min}} \tag{12}$$

and for $r_{\min}$ follows

$$\boxed{r_{\min} = \frac{H_{object}}{H - R - H_{object}} \cdot \sqrt{x_0^2 + y_0^2}} \tag{13}$$

with

$\sqrt{x_0^2 + y_0^2} \equiv$ distance between wind-energy-converter and object.

The coordinates of the "smallest" 'forbidden' circle are $(X,Y)_{\min}$ with

$$\boxed{(X,Y)_{\min} = (x_0, y_0) + r_{\min} \cdot (\cos t, \sin t)}\ ,\ t \in [0,2p]\ . \tag{14}$$

To be sure, that this circle is great enough, we have to analyse the possible error which occurs with respect to the uncertainties of the formulas for the sun's position. For a deviation of approximately 0.1° /4/ concerning the calculated angles it is possible to determine the necessary correction for the radius $r_{\min}$.

For the real shadow S_r there is $\tan(h_r) = \frac{H - R}{S_r}$ and therefore we obtain

$$\Delta S_{r\pm} = \frac{H - R}{\tan(h_r \pm 0.1)} - \frac{H - R}{\tan(h_r)}\ . \tag{15}$$

If $\Delta S_r < 0$ the calculated $r_{\min}$ is great enough, but in the case of $\Delta S_r \geq 0$ the radius $r_{\min}$ has to be changed in $(r_{\min} + \Delta S_r)$. With $h_r \in [0°, 90°]$ the corrected radius for the 'forbidden' circle is

$$\boxed{r_{\min,corr} = \frac{H_{object}}{H - R - H_{object}} \sqrt{x_0^2 + y_0^2} + \frac{H - R}{\tan(h_r - 0.1°)} - \frac{H - R}{\tan(h_r)}}. \tag{16}$$

Example: $H - R = 100m \quad \Rightarrow$

a) $h_r = 60° \quad \Delta S_r = (57.97 - 57.74)m = 0.23m$
b) $h_r = 30° \quad \Delta S_r = (173.9 - 173.2)m = 0.70m$
c) $h_r = 15° \quad \Delta S_r = (375.8 - 372.2)m = 3.6m$

The transformation for the shadow of the Rotor Hub is

$$(0,0,H) \rightarrow \frac{H}{\tan(h)}(\sin a, \cos a)\ , \quad a \in [-p, p]\ . \tag{17}$$

This position could be everywhere in the area around the Wind-Energy-Generator except in the range of the angle θ as shown in Figure 3. Using the general transformation rule for the shadow

$$(x, y, z) = (x, y, 0) + (0, 0, z) \rightarrow (x, y) + \frac{z}{\tan(h)}(\sin a, \cos a) \tag{18}$$

we finally obtain the result for the curvature of the rotator circle shadow :

$$(x_R, y_R, z_R) = (0,0,H) + R \cdot \underline{\underline{D}}(b)(\cos t_R, 0, \sin t_R)^T = (R\cos t_R \cdot \cos b, R\cos t_R \cdot \sin b, H + R\sin t_R)$$

$\rightarrow$

$$\boxed{(x_S, y_S) = R(\cos t_R \cdot \cos b, \cos t_R \cdot \sin b) + \frac{H + R\sin t_R}{\tan(h)}(\sin a, \cos a)} \tag{19}$$

With these formulas it is possible to predetermine the times and conditions, when the shadows of the rotating blades of a Wind-Energy-Generator will reach and overlap the 'forbidden' area. So we have valuable information to build up a control system to avoid the shadowing with minimum efficiency losses.

Finally the formula for the determination of the shadow-overlap with the restricted area could be worked out as

$$\boxed{\begin{array}{l} (x_S, y_S) = (X, Y)_{\min} \Leftrightarrow \\ R(\cos t_R \cdot \cos b, \cos t_R \cdot \sin b) + \frac{H + R\sin t_R}{\tan(h)}(\sin a, \cos a) = (x_0, y_0) + r_{\min} \cdot (\cos(t), \sin(t)) \end{array}}\ . \tag{20}$$

<u>Preview:</u>

These results enable us to develop a control system which avoids the casting of shadows on appointed regions. This could be done with minimum efficiency losses, because a stop of the rotation follows only if it is really necessary. Perhaps a manipulation of the orientation of the rotor circle may be sufficient. Naturally, a measurement of the insolation ought to be done. Depending on this the control system has to be active or inactive.
The orientation of the rotor circle relative to the north is the only parameter in the formula which has also to be measured, the others are constants including the date and time. It is planned to work out a computer program to achieve these possibilities and to propose some ideas for control system concepts.

Conclusion:

A mathematical formula is worked out, which could be the foundation of the development of a control system to avoid temporary casting of shadows from a Wind-Energy-Converter causing trouble with flickering daylight.

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