Electric Vehicles as a Topic for Applied School Mathematics

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Abstract

For the young generation it is indispensable to be concerned itself with the environmental consequences of the extensive usage of fossil fuels and to become familiar with renewable energies. Especially the topic of transport and traffic technology must be integrated in a respectively education of our children, and this can even be done advantageously for classroom teaching.
It seems to be particularly suitable to treat future energy issues in mathematics education; however, there is a great lack of respective teaching material. Thus a didactic concept in respect to content and structure of mathematical problems allowing their direct and broad usage in classroom has been developed. Some problems following this concept are presented in this contribution.

Keywords: education, energy consumption, emissions, power, modelling.

1 Introduction and Motivation

One of the most effective methods to achieve a sustainable change of our momentary existing power supply system to a system mainly based on renewable energy conversion is the education of our children. Especially the young generation would be more conflicted with the environmental consequences of the extensive usage of fossil fuels. For our children it is indispensable to become familiar with renewable energies, because the decentralised character of this future kind of energy supply requires surely more personal effort of everyone.
It would not be possible to build up a more effective, rational, and as far as possible renewable future energy supply system and structure, without a consequent integration of the *transport and traffic* technology. Most important for this is a complex and interconnected view of these problems as one whole undividable unit.
In comparison to the parental education, the public schools give the possibility of a successful and especially easier controllable contribution to this theme. This can even be done advantageously for classroom teaching, as realistic and attractive contents have a particular motivating effect on students. In addition to that, a contribution to interdisciplinary teaching would be given, which is a significant educational method, demanded by school curricula [1]. Regarding the fact, that in Germany not all students participate at technical oriented lessons in a comparable proportion, it seems to be especially suited to treat this topic in mathematics education for this purpose.
In addition this would be quite profitable for mathematics education itself, as "the application of mathematics in contexts which have relevance and interest is an important means of developing students' understanding and appreciation of the subject and of those contexts" [2, para F1.4]. However, although mathematics curricula demand application-oriented mathematics education, this not only in Germany [3, p.110], there is a great lack of mathematical problems suitable for school lessons [4, p.251]. Especially there is a need of mathematical problems concerning environmental issues that are strongly connected with future energy issues. An added problem is, that the development of such mathematical problems affords the co-operation of experts in future energy matters with their specialist knowledge and mathematics educators with their pedagogical content knowledge.

2 Didactical Concept

In such a co-operation the authors created first series of problems for the secondary mathematics classroom that will be completed by further ones. The cornerstones of the underlying didactical concept are:

- The problems are chosen that way, that the needed mathematical contents in order to solve them are part of mathematics school curricula.
- Advantageously every problem should concentrate on a special mathematical topic, such that it can be integrated in an existing teaching unit; as project-oriented problems referring to several mathematical topics are seldom picked up by teachers.
- The problems should not afford special knowledge of teachers concerning future energy issues and especially physical matters. For this reason all nonmathematical information and explanations concerning the problem's foundations are included in separated text frames.
- By going on this way information in respect to future energy issues is provided for both, teachers and students, helping them to concern themselves with the topic.

This didactical concept was first published by the authors at the '12. Internationales Sonnenforum 2000' in Freiburg [5], and meanwhile continued by several presentations in conferences respectively teacher education events ([6], [7], [8], [9], [10], [11], [12]), with much positive reactions. The authors are grateful for the broad support they earned and the valuable hints and materials they received with regard to the development of further problems.

3 Examples for Mathematical Problems

This contribution provides in the following further examples of mathematical problems concerning future energy issues. The problems presented here deal with the topic 'Electric Vehicles'. They are suitable for lessons in secondary schools. (Some of the data used can be found in [13], [14].)
The German version of the problems, including extensive explanations, and their solutions can be downloaded at the following internet address: http://www.math-edu.de under the topic 'Anwendungen'.

3.1 Example 1: The Problem of the CO_2 - Emission

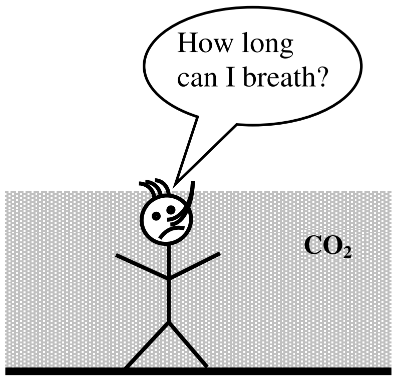
This is an inter-disciplinary problem linked to the subjects of mathematics as well as chemistry, physics, biology, geography, and social sciences. Nevertheless, it may be treated already in lower secondary class rooms. In respect to mathematics the *conversion of quantities* is practised, knowledge of *rule of three* and *percentage calculation* is required. The *amount of the annually in Germany produced* CO_2, especially also for the purpose of transport and traffic, is illustrated vividly, so that students become aware of it.

Info:

In Germany, each inhabitant produces annual averaged nearly 13 t CO_2 (Carbondioxid). Responsible for this emission into the atmosphere are combustion processes (for example from power plants or vehicle combustion motors).

Assume now, this CO_2 would build up a gaseous layer, which stays directly above the ground.

a) Which height would this CO_2-layer reach in Germany after one year?



Hints:

- Helpful for your calculation is knowledge from chemical lessons. There you learn, that amounts of material could be measured with the help of the unit 'mole'. *1 mole* CO_2 *weights 44 g and takes a volume of 22,4 l*, under normal standard conditions (pressure 1013 hPa and temperature 0°C). With these values you can calculate approximately.
- You will find the surface area and amount of habitants of Germany in a lexicon.

Help: Find the answers of the following partial questions in the given order.

i) How many tons CO_2 are produced in Germany in total every year?
ii) Which volume in l takes this amount of CO_2 ? (Regard the Hint!)
iii) How many m^3 CO_2 are therefore annually produced in Germany? Express this in km^3!
iv) Assume, the annually in Germany produced CO_2 would cover directly the ground as a lowest gas layer, which height would it have?

Info:

- In Germany the amount of waste is nearly 1 t for each habitant (private households as well as industry) every year, the averaged produced amount of CO_2 per habitant is therefore 13 times of this.
- The CO_2, which is produced during combustion processes and emitted into the atmosphere, distributes itself in the air. One part will be absorbed by the plants with the help of the photosynthesis, a much more greater part goes into solution in the oceans water. But the potential of CO_2 absorption is limited.
- 20% of the total CO_2-Emissions in Germany came in the years of the 90th solely from the combustion engines of the traffic activities.

b) Which height would the CO_2-layer over Germany take, if this layer results only from the annual emissions from individual vehicles? How many km^3 CO_2 are this?

3.2 Example 2: Energy Consumption

This problem can be treated in lessons of *trigonometry*. Its solving requires knowledge of *rule of three*. The problem makes clear the *dependence of an automobile's energy consumption* from the distance-height-profile, the moved mass and the velocity.

Tim and Lisa make a journey through Europe. Just before the frontier to Luxembourg their fuel tank is empty. Fortunately they have a reserve tank filled with 5 l fuel. „Let's hope it will be enough to reach the first filling station in Luxembourg. There, the fuel is cheaper than here“, Tim says. „It would be good if we have an exactly description of the route, than we would be able to calculate our range“, answers Lisa.

Info:

- In order to drive, the resisting forces are to overcome. Therefore a sufficient *driving force* F_{drive} is needed. For an average usual car, the law for this force (in N) is given by the following formula:
  $F_{drive} = (0{,}2 + 9{,}81 \cdot \sin\alpha) \cdot m + 0{,}3 \cdot v^2$ if $F_{drive} \geq 0$,
  with m as the moving mass (in kg), it means the mass of the vehicle, passengers and packages, v the velocity (in $\frac{\text{m}}{\text{s}}$) and α the angle relative to the horizontal line. α is positive for uphill direction and negative in the downhill case (Fig. 1).
- The energy E (in Nm), which is necessary for driving, can be calculated in cases of a constant driving force by:
  $E = F_{drive} \cdot s$, with s as the really driven distance (in m).
- The primary energy, consisting in the fuel, amounts about 9 kWh for each l fuel. (kWh is the sign for the energy unit 'kilowatt-hours'; it is 1 kWh = $3\,600\,000$ Nm).
- The efficiency concerning average driving conditions of usual combustion engines in cars is nowadays in-between 10% to 20%, this means only 10%-20% of the primary energy in the fuel are available to generate the driving forces.

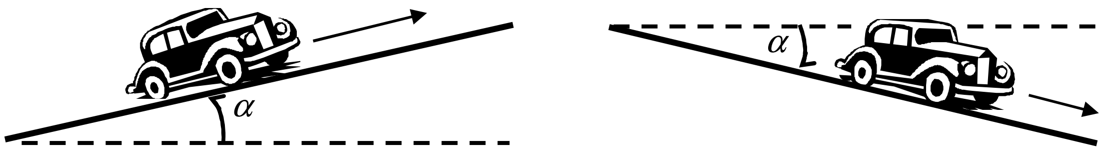


Figure 1: Definition of the angle α

The distance, which Tim and Lisa have to drive to the first filling station in Luxembourg can be approximately given by a graphical representation like that one given in Fig. 2. (Attention: think of the different scaling of the co-ordinate axis.) The technical data sheet of their vehicle gives the information of an empty weight about 950 kg. Tim and Lisa weight together ca. 130 kg, their packages nearly
170 kg. The efficiency of the engine can be assumed to be ca. 16%.

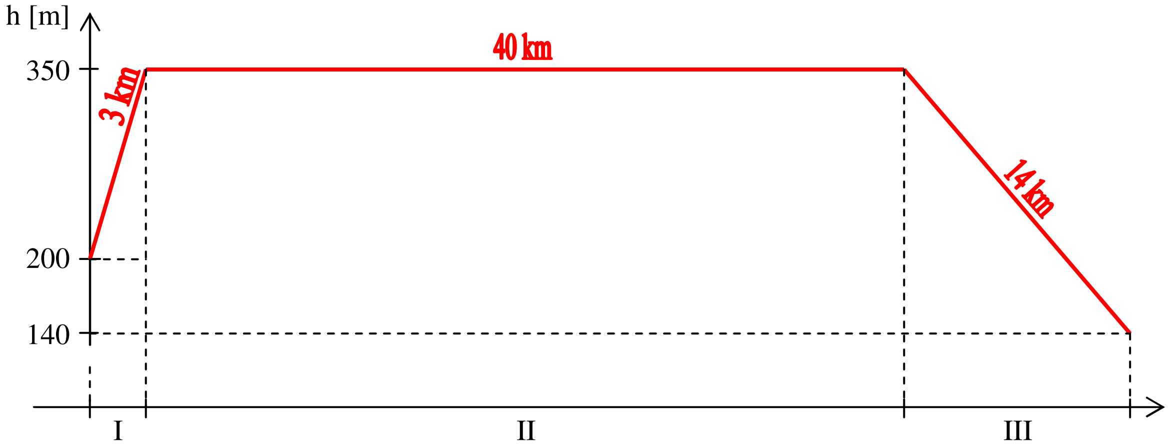


Figure 2: Distance-Height-Diagram to the next filling station (h is the height above mean sea level)

a) Can Tim and Lisa take the risk, not to search a filling station before the frontier?

   Assume at first, the speed they use is 100 $\frac{\text{km}}{\text{h}}$.

b) Would Tim and Lisa have less problems, if they have only 50 kg packages instead of 170 kg?

c) Would the answer to a) change, if Tim and Lisa choose their speed only to be 50 $\frac{\text{km}}{\text{h}}$?

Help to a):

i) Regard, the speed in the formula for F_{drive} has to be measured in $\frac{\text{m}}{\text{s}}$.

ii) The value for $\sin\alpha$ can be calculated with the information given in Fig. 2.

iii) Determine for each section the force F_{drive} and the needed energy. The distance s has to be measured in m. Convert the energy from Nm in kWh.

iv) Determine the total energy which is needed for the whole distance as a sum over the three different sections.

v) How many kWh energy to drive are given by the 5 l reserve fuel. Regard the efficiency of the motor.

3.3 Example 3: Forces, Energy and Power

This is a problem that can be treated in higher secondary mathematics education, in the frame of *differential and integral calculus*. By this problem the *dependence of an automobile's power and energy consumption* from the distance-height-profile, the moved mass and the velocity is shown.

Kay has a new electric vehicle, for which he is very proud of. He wants to drive with his girlfriend Ann from the disco at home. Ann slanders: „With this car you never reach over the hill to my home!" „I make a bet", says Kay.

The distance-height-characteristic of the street from the Disco (D) to the house (H), in which Ann lives, is shown in Fig. 3, and it can be described by the following function:

$$h(x) = \frac{1}{1\,000} \cdot (-4x^2 + 104x + 300) \text{ mit } x \in [0;\ 20],$$

whereas x and h are measured in kilometres [km].

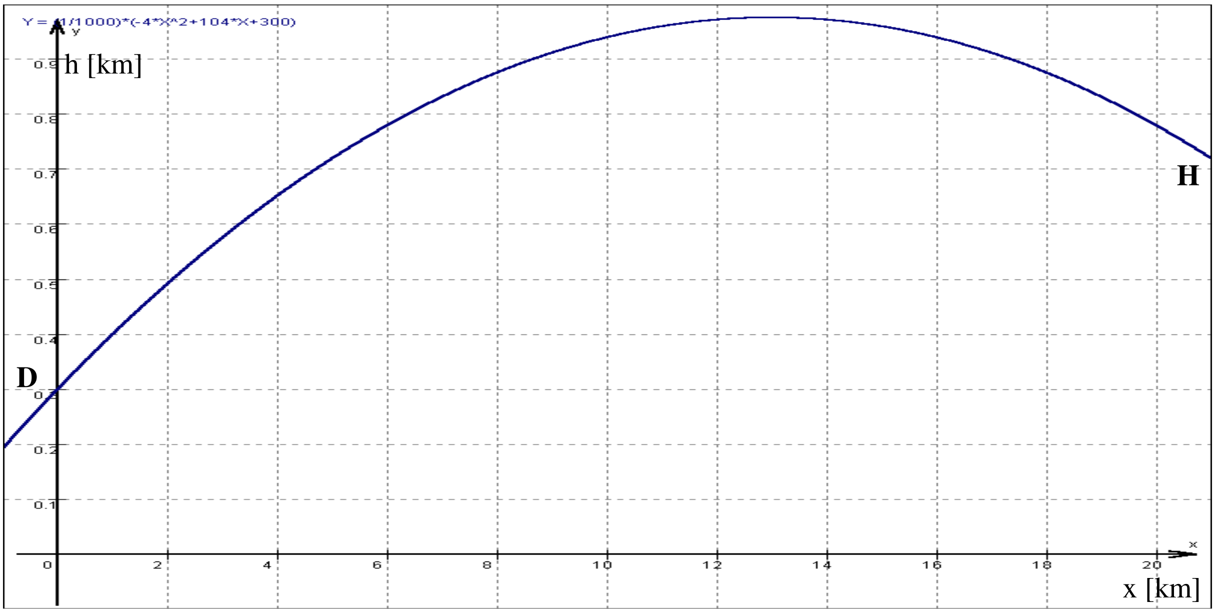


Figure 3: Distance-Height-Diagram between D and H

a) Show, that it is possible to calculate the real distance s depending of a given height function h over the interval $[x_1, x_2]$ with the help of the following formula:

$$s = \int_{x_1}^{x_2} \sqrt{1 + (h'(x))^2}\, dx .$$

Help: Regard the rectangular triangle as shown in Fig. 4.

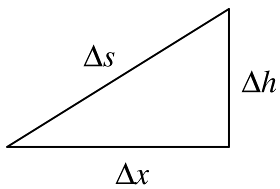


Figure 4: Geometrical correlation

b) How long is the real distance, which Kay has to drive from the Disco to the house, where Ann is living?

Help: Show that

$$\int \sqrt{1 + x^2}\, dx = \frac{1}{2}(x\sqrt{1 + x^2} + \ln(x + \sqrt{1 + x^2})) + \text{const.}$$

with the help of the deviation of $\frac{1}{2}(x\sqrt{1 + x^2} + \ln(x + \sqrt{1 + x^2}))$.

c) α means the angle of the tangent at the curve of h at the point x_0 with the x-axis. Prove that

$$\sin\alpha = \frac{h'(x_0)}{\sqrt{1+(h'(x_0))^2}}.$$

(Regard, that $h'(x_0) = \tan\alpha$.)

d) Assume, Kay wants to drive with a constant speed of $110\,\frac{\text{km}}{\text{h}}$.

Determine the necessary driving power at the top of the hill (maximum of h) and at the points with $h(x) = 0{,}4$ and $h(x) = 0{,}8$.

For this purpose you need the following data: Kay's electric vehicle has an empty weight of 620 kg, Kay and Ann weight together nearly 130 kg.

Info:

- In order to drive, the resisting forces are to overcome. Therefore a sufficient *driving force* F_{drive} is needed. For an average usual car, the law for this force (in N) is given by the following formula:
  $F_{drive} = (0{,}2 + 9{,}81\cdot\sin\alpha)\cdot m + 0{,}3\cdot v^2$ if $F_{drive} \ge 0$,
  with m as the moving mass (in kg), it means the mass of the vehicle, passengers and packages, v the velocity (in $\frac{\text{m}}{\text{s}}$) and α the angle relative to the horizontal line. α is positive for uphill direction and negative in the downhill case (Fig. 5).
- The driving power P (in $\frac{\text{Nm}}{\text{s}}$), which is needed to hold the constant speed, can be calculated with the product of the driving force (in N) with the velocity (in $\frac{\text{m}}{\text{s}}$): $P = F_{Antrieb}\cdot v$.
  The power P is to be measured with the unit [kW], with: $1\text{ kW} = 1\,000\,\frac{\text{Nm}}{\text{s}}$.

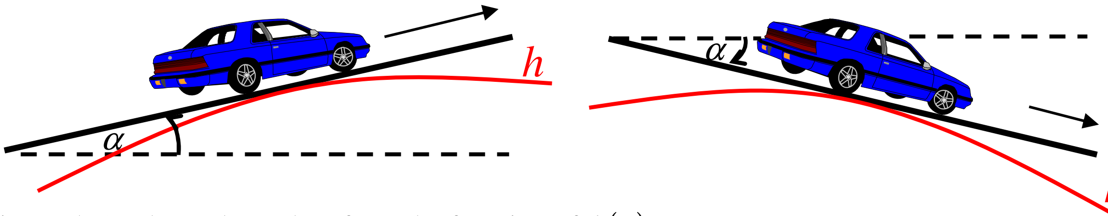


Figure 5: Angle α dependant from the function of $h(x)$

e) Kay's electric vehicle has a nominal power of 25 kW. Is it possible for him to bring Ann home?

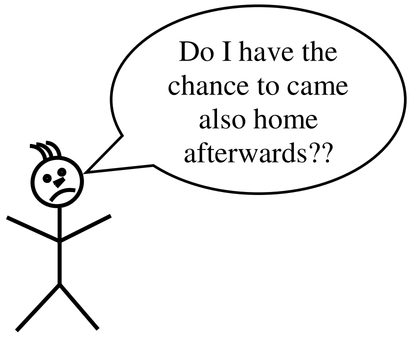
f) Determine the driving energy which has to be consumed for the route from the disco to Ann's home. Assume that Kay drives uphill with a speed of $80\,\frac{\text{km}}{\text{h}}$ and downhill with a speed of $110\,\frac{\text{km}}{\text{h}}$.

(Attention: Because of $h(x)$ as well as x are expressed in km in the function equation of h, the resulting energy referring to the following equation is obtained in $\text{N}\cdot\text{km} = 1\,000\text{ Nm}$.)

Info:

- The pure driving energy E (in Nm), which is necessary for driving, can be calculated as:
  $$E = \int_0^{s_0} F_{drive}\,ds = \int_0^{x_0} F_{drive}\sqrt{1+(h'(x))^2}\,dx,$$ with s as the really driven distance (in m).
- A usual unit to measure the energy E is kilowatt-hour [kWh]. It is: 1 kWh = 3 600 000 Nm.

g) The actually charged electrical energy in the batteries of Kay's vehicle is 6 kWh. The driving efficiency of his electrical vehicle is nearly 70%, this means only 70% of the stored energy can be used for driving.
Is the charging status of Kay's batteries sufficient to bring Ann to her home, regarding the assumptions under f)?



4 Final Remarks

This contribution is a further step to integrate the important issues of future energy in curricula of public schools. For this purpose several initiatives have been started in Germany, supported and co-operated by the 'Deutsche Gesellschaft für Sonnenergie e.V. (DGS)', the German section of the ISES (International Solar Energy Society). The wide spectrum of respective activities going on has been presented amongst others also on the 3rd Solar Didactica within the Solar-Energy World Exposition. This event took place under the patronage of the German minister for education and research E. Bulmahn, expressing thus the great interest and importance devoted by politicians.

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