# WIND ENERGY IN SECONDARY MATHEMATICS CLASSROOMS

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ABSTRACT: Following the general didactical concept published first by the authors at the '12. Internationales Sonnenforum 2000' in Freiburg, meanwhile continued by several presentations with much positive reactions, this contribution presents further examples of mathematical problems concerning future energy issues. One of the most important cornerstones to build up a more effective, rational, and as far as possible renewable future energy supply system and structure is the use of *wind energy*. The problems presented here deal with the very important and indispensable topic 'Wind Energy Conversion'. They are suitable for lessons in secondary schools and include extensive explanations. This contribution is a further step to integrate such important themes in curricula of public schools. For this purpose, we have started an initiative supported and cooperated by the 'Deutsche Gesellschaft für Sonnenergie e.V. (DGS)', the German section of the ISES (International Solar Energy Society).
Keywords: Education, Renewable Energy, Modelling

## 1 INTRODUCTION AND MOTIVATION

One of the most effective methods to achieve a sustainable change of our momentary existing power supply system to a system mainly based on renewable energy conversion is the education of our children. Especially the young generation would be more conflicted with the environmental consequences of the extensive usage of fossil fuels. For our children it is indispensable to become familiar with renewable energies, because the decentralised character of this future kind of energy supply requires surely more personal effort of everyone.

In comparison to the parental education, the public schools give the possibility of a successful and especially easier controllable contribution to this theme. This can even be done advantageously for classroom teaching, as realistic and attractive contents have a particular motivating effect on students. In addition to that, a contribution to interdisciplinary teaching would be given, which is a significant educational method, demanded by school curricula [1]. Regarding the fact, that in Germany not all students participate at technical oriented lessons in a comparable proportion, it seems to be especially suited to treat this topic in mathematics education for this purpose.

In addition this would be quite profitable for mathematics education itself, as "the application of mathematics in contexts which have relevance and interest is an important means of developing students' understanding and appreciation of the subject and of those contexts" [2, para F1.4]. However, although mathematics curricula demand application-oriented mathematics education, this not only in Germany [3, p.110], there is a great lack of mathematical problems suitable for school lessons [4, p.251]. Especially there is a need of mathematical problems concerning environmental issues that are strongly connected with future energy issues. An added problem is, that the development of such mathematical problems affords the co-operation of experts in future energy matters with their specialist knowledge and mathematics educators with their pedagogical content knowledge.

## 2 DIDACTICAL CONCEPT

In such a co-operation the authors created first series of problems for the secondary mathematics classroom that will be completed by further ones. The cornerstones of the underlying didactical concept are:

- The problems are chosen that way, that the needed mathematical contents in order to solve them are part of mathematics school curricula.
- Advantageously every problem should concentrate on a special mathematical topic, such that it can be integrated in an existing teaching unit; as project-oriented problems referring to several mathematical topics are seldom picked up by teachers.
- The problems should not afford special knowledge of teachers concerning future energy issues and especially physical matters. For this reason all nonmathematical information and explanations concerning the problem's foundations are included in separated text frames.
- By going on this way information in respect to future energy issues is provided for both, teachers and students, helping them to concern themselves with the topic.

This didactical concept was first published by the authors at the '12. Internationales Sonnenforum 2000' in Freiburg [5], and meanwhile continued by several presentations in conferences respectively teacher education events ([6], [7], [8], [9], [10], [11], [12], [13], [14], [15], [16], [17]), with much positive reactions. The authors are grateful for the broad support they earned and the valuable hints and materials they received with regard to the development of further problems.

## 3 EXAMPLES FOR MATHEMATICAL PROBLEMS

This contribution provides in the following further examples of mathematical problems concerning future energy issues. The problems presented here deal with the topic 'Wind Energy Conversion'. They are suitable for lessons in secondary schools. (Some of the data used can be found in [18], [19], [20], [21].) With respect to the limited space in this paper, this presentation is focused on the basic structure of the problems, in order to give an impression of the didactical concept and the general principles.

The German version of the problems, including extensive explanations, and their solutions can be downloaded at the following internet address: http://www.math-edu.de under the topic 'Anwendungen'.

### 3.1 Example 1: Wind Energy Development

- This problem requires the usage and interpretation of data and statistics.

*Info:*

At the end of 1990 the wind energy converters in Germany had a total installed nominal power of 56 MW. At the end of 2000 this amount increased to a total of 6113 MW.

The following table shows the development of the new installed wind power in Germany between 1991-2000 [21].

**Table 1:** Development of new installed Wind Energy in Germany

| Year | Number of new installed wind energy converters | Total of new installed nominal power | Total of nominal power |
|---|---|---|---|
| 1991 | 300 | 48 | |
| 1992 | 405 | 74 | |
| 1993 | 608 | 155 | |
| 1994 | 834 | 309 | |
| 1995 | 911 | 505 | |
| 1996 | 804 | 426 | |
| 1997 | 849 | 534 | |
| 1998 | 1010 | 733 | |
| 1999 | 1676 | 1568 | |
| 2000 | 1495 | 1665 | |

a) Fill in the missing data in the $4^{th}$ column.

b) Show the development of the annual new installed power respectively of the total annual power in graphical representations.

c) Regarding only the data of the years 1991-1998, which development in respect to the installation of new wind power in Germany could be expected in your opinion? Give a well-founded answer!

Compare your answer with the real data given for 1999 and 2000 and comment it.

What is your prognosis for 2005? Why?

d) Calculate out of the data in table 1 for each of the years 1991-2000 the average size of new installed wind energy converters in kW.

Show the respective development graphically and comment it.

Can you dare a prognosis for the average size of a wind energy converter that will be installed 2010?

### 3.2 Example 2: Wind Energy Converter and Geometry

- This problem can be treated in lessons of *geometry, especially calculations of circles*. The *conversion of quantities* is practised.

*Info:*

The nominal power of a wind energy converter depends upon the rotor area A with the diameter D as shown in figure 1 below.

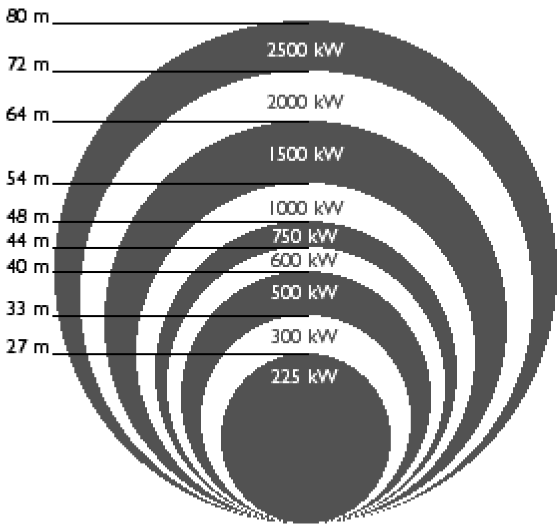


**Figure 1:** Nominal Power related to the Rotor Area [20]

a) Interpret the meaning of the figure above. Show the dependence of the nominal power of wind energy converter on the rotor diameter and respectively on the rotor area by graphs in co-ordinate systems.

b) Find the formula, which gives the nominal power of wind energy converter as a function of the rotor area and respectively of the rotor diameter.

c) Which rotor area would you expect to need for a wind energy converter with a nominal power of 3 MW? Which length should the rotor blades have for this case?

*Info:*

In Germany, wind energy converter produce in average their nominal power during approximately 2000 hours of a year, with respect to sufficient wind energy conditions.

d) Calculate the average amount of energy in kWh, which would be produced by a wind energy converter with a nominal power of 1,5MW during one year in Germany.

*Info:*

An average household in Germany consumes nearly 4000kWh electrical energy per year.

e) How many average private households in Germany could be theoretically be supplied with electrical energy by a 1,5MW wind energy converter? Why do you think, that this could only be a theoretical calculation?

f) How many percentage of the year would in average be covered with a nominal power rate supply of a wind energy converter in Germany?

g) Assume, the nominal power of a 600kW energy converter would be reached at a wind speed of 15m/s, measured at the hub height. How many km/h are this? How fast are he movements of the tips of the blades, if the rotation speed is 15/min. Give the solution in m/s and km/h respectively. Compare the result with the wind speed.

### 3.3 Example 3: Betz' Law and Differentiation

- This problem can be treated in lessons of *differentiation and the determination of local extreme values*.

*Info:*

2% of the insolated solar energy is converted to kinetic energy of the air molecules. In combination with the earth rotation, this results in a wind production. The kinetic energy of an air mass $\Delta m$ is $E = 1/2 \cdot \Delta m \cdot v^2$. With the help of the density of air $\rho = 1{,}2\text{g/l}$ and the relation $\Delta m = \rho \cdot \Delta V$ with the volume element $\Delta V$, the kinetic energy can be written as $E = 1/2 \cdot \rho \cdot \Delta V \cdot v^2$. The power is defined as the ratio of energy to time as $P = E/\Delta t$.

a) A wind volume $\Delta V$ flows through a rotor area $A$ and needs the time $\Delta t$ for the distance $\Delta s$. Therefore the speed is $v = \Delta s/\Delta t$. Determine the general formula for the volume element, which passes the rotor area $A$ during the time interval $\Delta t$ as a function of the wind speed.

b) Give the formula for the amount of wind power $P_{wind}$, which passes through the rotor area $A$ in dependence of the wind velocity.

Show, that the power increases with the third power of the wind velocity.

*Info:*

A rotor of a wind energy converter with the area $A$ slows down the incoming wind speed from $v_1$ in front of the rotor to the lower speed $v_2$ behind the rotor. The wind speed in the rotor area itself can be shown to be the average of $v_1$ and $v_2$ as $\bar{v} = (v_1 + v_2)/2$. The converted power is than given by:

$$P_c = P_1 - P_2 = \frac{1}{2} \cdot \rho \cdot \frac{\Delta V}{\Delta t} \cdot \left(v_1^2 - v_2^2\right).$$

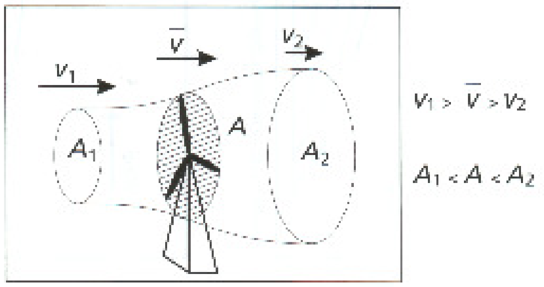


**Figure 2:** Wind Flow through the Rotor Area

c) Express the formula for the determination of $P_c$ in dependence of $A$, $v_1$ and $v_2$.

d) Describe the converted power $P_c$ in c) as a function of the variable $x = v_2/v_1$ and $v_1$.

*Info:*

The efficiency of a wind energy converter (power coefficient) is defined as the ratio of the converted power to the wind power input as $c_P = P_c/P_{wind}$.

e) Express the power coefficient as a function of the variable $x = v_2/v_1$. Draw the graph of this function in dependence of $x$. Notice: It is $x \in [0,1]$, why?

f) Determine the value $x_{\max}$ which corresponds to the maximum value of the power coefficient, the so-called Betz' efficiency. This means the value for $x = v_2/v_1$, which gives the best energy conversion.

## 4 FINAL REMARKS

This contribution is a further step to integrate the important issues of future energy in curricula of public schools. For this purpose several initiatives have been started in Germany, supported and co-operated by the 'Deutsche Gesellschaft für Sonnenergie e.V. (DGS)', the German section of the ISES (International Solar Energy Society). The wide spectrum of respective activities going on has been presented amongst others also on the 3rd Solar Didactica within the Solar-Energy World Exposition. This event took place under the patronage of the German minister for education and research E. Bulmahn, expressing thus the great interest and importance devoted by politicians. The last event promoting future energy issues in secondary class rooms in Germany, the 5th Solar Didactica, just took place in Berlin and showed again the continuous interest on these topics.

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