Mathematical Optimisation Strategies for PV-Hybrid Systems

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Introduction

Unfortunately the term optimisation is very often used in different meanings, especially from engineers in practice, when they work on something solely with the aim to reach a better operational status. In many cases, this is not an optimisation in a strong mathematical sense. In order to find a real optimum for more or less complex hybrid energy supply systems, a well structured optimisation process with respect to mathematical correctness would be helpful. This method gives also the possibility of an evaluation of the whole system with a higher degree of objectivity.

Abstraction of Hybrid Systems

In order to describe an optimisation problem, especially for a PV-hybrid system, it is necessary to find a schematic presentation of these kind of systems with all optional different realisation possibilities, which allows a transformation of the technical meanings and their interconnections into abstract mathematical formulations with a one-to-one correspondence. The special fundamental schematic, as shown in figure 1 was originally created with this intention [1]. This schematic has the structure of a quadratic matrix, with three rows and three columns. The columns represent the converter, storage and consumer, whereas the rows point out the three kinds of used energy like electrical, thermal and chemical energy (e.g. material flow). A special and important component is given by the control system, although it is responsible for the whole functioning, it needs also an energy supply.

The combination of the different components in figure 1 should be understood as optional (not necessarily implemented) realisation possibilities, therefore all thinkable PV-hybrid systems can be principally represented by this scheme. It doesn't matter, how complex a system is. In a simple case, the system can be a small single unit for rural electrification, but it represents also complex interconnected systems even up to energy supply systems for a whole country. In complex cases, the single components represent matrixes for themselves with more differentiated interconnecting lines and figure 1 serves as an abstraction for the main energy fluxes.

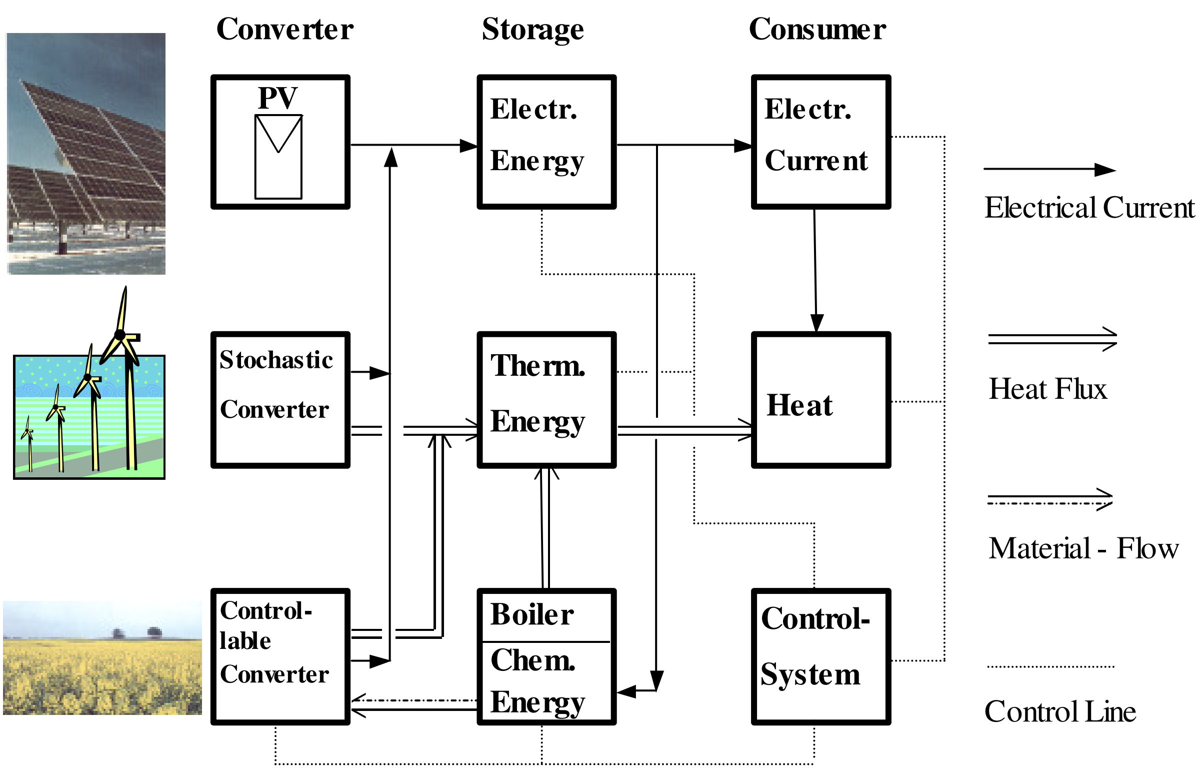

Figure 1 Fundamental Systematic of PV-Hybrid Systems

With respect to the need of electrical as well as thermal energy, the above figure symbolises complex hybrid system possibilities with combined heat and power. The final goal to realise a fully renewable energy supply implies necessarily the integration of biomass converter [2], [3]. In contrast to a PV-generator or a wind energy converter, biomass can be used in a controllable way.

For this reason, it is helpful to differentiate between stochastically and controllable converter. This has a very important effect on the calculation principles which are to be used for optimisation purposes. Because the system as a whole has to be controllable, there is naturally a strong interconnection to the dimensioning principles.

Formulation of Optimisation Problems

In everyday language, the term optimisation is often used in the sense of improvement without specifying the means. To find a real optimum, we have to search the best and not only a better solution and to employ a mathematical model in order to find a solution [4]. Therefore the formulation of an optimisation problem always consists of

- a mathematical model of the problem which has to be optimised,
- an objective function,
- additional constraints.

A general optimisation problem is given, if the following list is valid

$$\begin{cases} H \text{ - Space} \\ S \subseteq H \text{ - Subspace of } H \\ f : S \to \mathbb{R} \text{ - Function, which is difined on } S \end{cases} \quad (1)$$

with $\mathbb{R}$ as the space of real numbers. The optimisation problem (OP) can be formulated as a search for

$$(OP)\begin{cases} \text{a number } f^* \in \mathbb{R} \text{ and} \\ \text{an element } x^* \in S, \text{ so that} \\ f^* = f\left(x^*\right) \le f\left(x\right), \ \forall x \in S \end{cases} \quad (2)$$

with f as objective function, S the restricted domain, f^* as optimum and x^* as optimal solution. An $x \in S$ is called possible solution.

Constraints can be present or not, they can be equality and/or inequality constraints. Without constraints, for so-called free problems, there is $S = H$. The objective function and the constraints can be linear or non-linear [5]. Variables can be discrete (e.g. $x \in \mathbb{N}^n$ or $x \in \mathbb{Z}^n$) or continuous (e.g. $x \in \mathbb{R}^n$).

The space of the degrees of freedom can be finite or infinite dimensional. In the case of a finite dimensional space, we have to determine a solution like $x^* \in \mathbb{R}^n$. For time dependant problems, the space of the degrees of freedom is infinite dimensional. The solution $x^*(t)$ is a function of time, i.e. $x^*(t): \mathbb{R} \to \mathbb{R}^n$.

Constraints can often be formulated with the help of functions like

$$\begin{aligned} g(x) &\le 0 \quad (\text{inequality constraints}) \\ h(x) &= 0 \quad (\text{equality constraints}). \end{aligned} \tag{3}$$

Hence the problem may be described by three functions

$$\begin{aligned} f &: \mathbb{R}^n \to \mathbb{R}, \\ g &: \mathbb{R}^n \to \mathbb{R}^{m_g}, \\ h &: \mathbb{R}^n \to \mathbb{R}^{m_h}. \end{aligned} \tag{4}$$

Here n is the number of scalar variables of the problem, m_g and m_h are numbers of (scalar) inequality and equality constraints. The difference $d = n - m_h$ represents the number of degrees of freedom and indicates how many variables can be chosen independently. In addition, the terms local and global optimisation are used. Local optimisation means the best solution in a subset of the possible local region around the given initial point, whereas global optimisation aims at finding the best of all possible solutions. These terms refer to the mathematical model and not to the problem itself which is represented by the reduced (i.e. simplified) model. In some cases, the accessible domain is already reduced by the modelling process itself.

Optimisation Strategies for Hybrid Systems

Fundamental dimensioning principles for PV-hybrid systems are already worked out, with respect to the combining effects of the characteristics of a PV-plant with an additional stochastic converter, for example like a wind turbine [1], and the addition of controllable biomass converter [2] especially for modern steam engines [3]. But for optimisation purposes, more detailed calculations are necessary, especially for time dependant simulations. In order to give the opportunity for flexible calculations, a manifold MatLab-Toolbox with computational programs for most of the possible components of renewable energy hybrid systems is under construction, to develop more and more basic subroutines for further complex numeric routines. These programs are designed as fundamental functions for different mathematical modelling purposes, and especially for optimisation processes. Each component has to be implemented as a mathematical model for itself. With respect to the stochastically behaviour of some converter and many of the loads, a special focus is given to the dynamical physical characteristics. The combination and interconnection of the single models of the components, leads to a complete mathematical model for the whole system. This system-model can principally be calculated with the help of the MatLab-subroutines and functions, added together to a main program. This program should also allow dimensioning as well as simulation calculations. With respect to the aim of a optimisation, the computational programs have to be as flexible as possible and of universal character, because the search of the optimum is in most cases only possible with the help of numerical processes.

After a valuable mathematical model has been worked out, the next difficulty is to find a useful objective function. The question for the objective function is, what has to be optimised? Possible optimisation criteria could be aspects like maximum availability, maximal overall efficiency, minimal emissions, minimal costs and many others similar to these. What should be the best choice for the objective function? It would be alternatively helpful to define a combined objective function like the following

$$F(x)=\sum_{i=1}^{k}\xi_i \cdot f_i(x), \qquad \text{with} \qquad \sum_{i=1}^{k}\xi_i = 1, \tag{5}$$

and $0<\xi_i<1$ as weight parameters for the k different sub-objective functions f_i. With such a combined function F as an objective function, optimising strategies with respect to different interests may be possible in order to give a compromise.

The next problem is to specify the necessary restrictions, which lead to the constraints functions. First of all, important restrictions are given by the maximal capabilities of all the components. Other restrictions are determined by the load demands. Whereas some restrictions are well determined concerning their time dependant behaviour, there exist also stochastically determined constraints.

Especially the additional integration of constraint functions with respect to probabilistic variables, with continuous or discrete characteristics, leads to a significant increase in the degree of difficulty for solving such optimisation problems.

With respect to the extensive usage of biomass for the combined heat and power with a hybrid system like shown in figure 1, an other important aspect should be carefully added to the search of necessary constraints. Because the biomass needs a regional infrastructure to reach the hybrid system from the surrounding environment. It may be fatal for the near biosphere, if the planning process neglects the question of a sustainable balance with respect to the growth of the biomass. This means the integration of constraints from the regional material flow management.

Dependant from the kind of hybrid system, the components, its complexity, dimension, extension and location, it would possibly be necessary to extend the amount of possible restrictions regarding also to some aspects of environmental physics.

The mathematical modelling and optimisation of PV-hybrid systems can give a valuable insight to use the high potential of such theoretical strategies and methods to play an important role in the planning and automation philosophy of future concepts and constructions of decentralised energy supply systems.

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